# The Link Between Macroeconomic Factors and Style Returns

Qi Jessica Zhang[1], Peter Hopkins[2], Steve Satchell[3], Robert Schwob[4]

1. *Department of Economics, University of Cambridge*
2. *Style Research Limited, London*
3. *Department of Economics, University of Cambridge*
4. *Style Research Limited, London*

* *We are grateful for the data kindly provided by Professors Elroy Dimson and Paul Marsh from London Business School.*

## Abstract

Though the size premium and value premium have been well recognized, the risk-based explanations behind them have not been extensively exploited yet. This paper examines the economic nature of the Fama-French size and book-to-price factors and establishes a significant link between the style factors and macroeconomic state variables using two different approaches: (1) discrete state analysis, and (2) threshold regression. The results from these two methods support the same conclusions. Firstly, value and small caps have performed best in periods of higher GDP growth; secondly, there exists a positive relationship between unexpected inflation and the value premium, and a negative relationship between unexpected inflation and the size premium; thirdly, value and smaller stocks perform better when short term interest rates are low; finally, we find a positive relationship between the return premiums and the term spread.

JEL Classification: G11; G12; G15

## I. Introduction

The existence of the size premium and value premium has been widely recognized. The size premium implies that small stocks tend to have higher average returns than big stocks, whereas the value premium means that value stocks tend to have higher average returns than growth stocks. This phenomenon has drawn attention from a wide range of academics and practitioners. Merton (1973) found that, in a complete factor pricing model, additional factors, or state variables, are required to capture the risks due to changes in the investment opportunities set. It may well be the case that the size premium and value premium might result from non-diversifiable risk that is not captured by the standard capital asset pricing model.

It is well understood that asset prices react sensitively to economic news. Therefore, many researchers attempt to use macroeconomic variables as candidates for state variables that proxy for the investment opportunity set (Chen, Roll and Ross 1986, Ferson and Harvey 1991 etc). It follows from capital market theory that stocks returns should only be rewarded for their systematic risk exposure and no extra reward should be earned by bearing diversifiable risk. In particular, the co-movement of asset returns suggests the presence of underlying systematic risk. Therefore our task is to choose a set of appropriate economic state variables: GDP growth, inflation, short term interest rates, the term spread and the credit spread as sources of systematic asset risk; these choices follow from recent research, (see, inter alia, Liew and Vassalou, 1999; Vassalou, 2000; Kelly, 2003; Petkova, 2006; Aretz etc, 2007). In principle, as stock prices can be thought of as expected discounted future cash flows, any systematic variables affecting the future cash flow and discount rate would also influence the stock prices. One of the most important factors influencing cash flow is GDP growth, which would reflect the investment opportunities and therefore the level of cash flow. The positive and significant relationship between the return premia and GDP growth is identified in many papers (Liew and Vassalou (1999), Kelly (2003)). Also, changes in expected inflation will influence nominal expected cash flow as well as the nominal interest rate. On the other hand, a change in unexpected inflation is more likely to have a systematic effect and change the relative price. The discount rate is an average rate over time, changing with respect to the level and term structure of the yield curve. Thus the 3m T-bill rate, term spread and credit spread are all possible factors we will investigate.

Another substantial section of the finance literature focuses on stock characteristic-based factors. Return premia for size stocks and value stocks have been widely documented. Fama and French (1992, 1993, 1996, 1997) propose characteristic sorted book-to-market (HML) and size (SMB) portfolios as state variables in the sense of Merton's (1973) ICAPM, where SMB and HML are constructed from six size/book-to-price benchmark portfolios. SMB (Small Minus Big) is the return difference between small portfolios and big portfolios; HML (High Minus Low) is the return difference between high book-to-market portfolios and low book-to-market portfolios. A series of papers by Fama and French show that their three-factor model, which includes the excess market return, HML and SMB, can explain the cross-section of returns better than the CAPM. Ferson and Harvey (1991), Vassalou (2003), and Petkova (2005) showed that HML and SMB lose significance in the presence of other economic variables.

Our question is what is the risk-based explanation behind the HML and SMB factors? The aim of this paper is to examine the relationship between the HML and SMB portfolios and the macroeconomic variables. For the most part, can we relate the style based portfolio performance to macroeconomic variables and business cycle fluctuations? Can we identify the economic cycles potentially favorable to value firms or small firms? This is a field which has not yet been fully explored in the literature.

In this paper, we present a comprehensive analysis of the relationship between Fama and French factors and some fundamental macroeconomic factors. We will address these issues by focusing on the UK and US evidence. We use the concept of discrete state analysis to investigate the inter-relationship of SMB/HML and economic states. This methodology allows us to analyze how the portfolios perform under different economic environments. Threshold regression is also introduced so as to capture the possible non-linear relationship between the portfolio returns and macroeconomic variables. The results from two methods support the same conclusions. Firstly, value and small caps have performed best in periods of higher GDP growth – a result that accords with intuition, as one would expect economic growth to provide increased investment opportunities and lower the risk for distressed companies; secondly, there exists a positive relationship between unexpected inflation and the value premium, and a negative relationship between unexpected inflation and the size premium; thirdly, value and smaller stocks perform better when short term interest rates are low. Such firms will be more likely to suffer (compared with longer duration growth stocks) when short term rates are high; lastly, we find a positive relationship between SMB/HML and the term spread, suggesting a possible greater vulnerability of longer duration growth stocks to inflation and future uncertainty.

The remainder of this paper is organized as follows: Section II introduces the data and the possible relationships between style-based portfolios and macroeconomic variables; section II presents the empirical results by implementing discrete state analysis; section 4 applies the threshold regression to identify the non-linear effect; section 5 concludes.

## II. Data

### *II.i. Style Based Portfolios*

The London Share Price Database (LSPD) maintained at London Business School provides accounting information for all UK firms listed on the London Stock Exchange going back to 1955. The portfolio formation mechanism follows Black, Mao and McMillian (2008). We rank the stocks based on two criteria: market capitalization and the book-to-price ratio. SMB, and HML are constructed from six size/book-to-price benchmark portfolios. SMB (Small Minus Big) is the return difference between small portfolios and big portfolios. HML (High Minus Low) is the return difference between the high book-to-market portfolio and low book-to-market portfolio. The US style based portfolio (Fama-French benchmark factors) data series are simply downloaded from French’s website, with data back to 1926. Table 1 shows the basic statistics for HML and SMB portfolios in UK and US. Clearly both HML and SMB portfolios generate positive returns as expected.

*Table 1: Average and standard deviation of monthly returns on HML, SMB portfolios*

| | SMB | | HML | |
|---|---|---|---|---|
| Country | Mean | Std | Mean | Std |
| UK(from 1955 Sep) | 0.15% | 3.69% | 0.47% | 2.25% |
| US (from 1926 Sep) | 0.23% | 3.24% | 0.34% | 3.65% |

### *II.ii Macroeconomic Factors*

As mentioned previously, stock prices can be thought of as expected discounted future cash flows, so any systematic variables affecting the future cash flow and the discount rate should also influence stock prices. Table 2 summarizes the hypotheses between style-based portfolios and macroeconomic variables.

<u>*GDP growth rate*</u>

The quarterly seasonally adjusted real GDP data series for the UK are obtained from the Office of National Statistics, with data back to 1955 Q1. The GDP data for the US are downloaded from the Bureau of Economic Analysis, with a relatively longer series back to 1947 Q1. We compute quarterly growth rates of GDP as: $GDPgrowth_t = \ln(GDP_t) - \ln(GDP_{t-3})$. As unanticipated innovations in macroeconomic variables can capture the uncertainty about investment opportunities in the future, and influence stock returns, it is therefore important to specify the time-series dynamics of the state variables. In particular, considering that asset prices are forward looking, next quarter's GDP growth rate would be a more appropriate factor. Here we assume that the innovation in GDP growth rate is the difference between the GDP growth rate at time t and its moving average in the previous four quarters.

Typically, real economic growth coincides with increased investment opportunities for distressed firms, either because it is the increased investment opportunities that lead to growth or because increased growth lowers risk, making more projects available.

Therefore GDP growth rate is thought likely to have a positive relationship with the size premium and value premium.

*Inflation*

For the UK, quarterly retail price index data back to 1955 Q2 is available from the Bank of England. For the US, we can obtain the seasonally adjusted monthly consumer price index back to 1947 M1 from Bureau of Labor Statistics. Realized inflation is defined as the monthly first difference in the logarithm of the consumer price index: $realized\ I_t = \ln(CPI_t) - \ln(CPI_{t-1})$ . As suggested by Fama and Gibbons (1984), expected inflation is the difference between the current T-bill rate and its 12-month (or 4 quarters) moving average: $\exp ected\ I_t = Tbill_t - \sum_{j=1}^{12} Tbill_{t-j}$. In this paper, the unexpected inflation is calculated as the difference between realized and expected inflation.

A negative relationship between SMB and unexpected inflation is commonly assumed, suggesting that small firms suffer more in a high unexpected inflation environment. On the other hand, there is more likely to be a positive relationship between unexpected inflation and the value premium. High unexpected inflation implies market uncertainty and indicates that the central bank is likely to raise interest rates in the future and to discourage investment in longer duration firms. In addition, value firms usually are high dividend paying firms and appear to be good short term investments; growth firms pay low dividends and retain most of their profit for reinvestment purposes, thus providing for future earnings and dividend growth. Therefore in the high unexpected inflation environment, value firms ought to perform better than growth firms.

*Short term interest rate*

We choose the 3m T-bill rate as the proxy for the risk free interest rate, which conveys information about the level of the yield curve. The 3m T-bill rate for the UK is obtained from the Bank of England, with data back to February 1970. We obtained the US 3-month T-bill rate back to January 1934 from the Federal Reserve Statistical Release.

As value firms and small firms tend to be low duration companies with high leverage and cash flow uncertainty, high short term interest rates imply that value firms and small firms will be hit more badly, suggesting a negative relationship between the 3m T-bill and value portfolio returns.

*Term Spread*

For the UK, we get the 10 year government bond yield from the Bank of England, with data available from 1970 M2; for the US, the 10-year government bond data is available back to April 1953 from the Federal Reserve Statistical Release.

The term spread is a measurement of the slope of the yield curve. The term spread is defined as the yield difference between the 10year government bond and the 3m T-bill rate. We use this variable to capture the returns on long duration bonds. Hence, firms dependent on the long end duration will be more heavily hit by the increase of long term interest rates, suggesting a positive relationship between SMB/HML and term spread.

*Credit Spread*

The credit spread is calculated as the yield spread between a high yield bond and a 10 year government bond, as a proxy for risk premium. For the US, the Moody Baa corporate bond yield index is downloaded from Bloomberg with data back to April 1954.

*Table 2 Hypotheses between the SMB/HML and economic state variables*

| | GDP growth | Infl inno | 3m T-bill | Term spread | Credit Spread |
|---|---|---|---|---|---|
| SMB | Positive | Negative | Negative | Positive | |
| HML | Positive | Positive | Negative | Positive | |

## III Discrete State Analysis

We divided the period into three states of the world: when the variable is high (top 25%), neutral (middle 50%) or low (bottom 25%). To test the relationships between stock characteristic factors (HML and SMB) and macroeconomic variables, we implemented two different approaches by treating the variables as categorical variables and quantitative variables.

### *III.i. Coincidence matrix*

We treated all variables as categorical variables and tested whether they were independent or not using a chi-square independence test. By classifying the variables into three states: top 25% state, middle 50% and bottom 25%, we get a 3 by 3 matrix for each pair of variables $X_i$ and $X_j$ . For the chi-square test of independence, we list a contingency table where the probability of outcomes falling in each cell is recorded. Each individual cell is the intersection of each $X_i$ and $X_j$ event class. With that in mind, if the null hypothesis is that $X_i$ and $X_j$ are independent, then the probability of the intersection is equal to the product of the two associated marginal probabilities.

Below is an illustration of the chi-square independence test for two categorical variables. Table 3 displays the contingency table for two variables: SMB and GDP growth innovation for the US, where the numbers in each cell are observed probabilities and expected probabilities respectively. The numbers in the bracket are the expected probabilities, calculated as the simple product of the associated marginal probabilities.

*Table 3: Contingency table example: SMB and GDP growth innovation*

| Probability | | GDP growth innovation | | | |
|---|---|---|---|---|---|
| | | Top 25% | Middle 50% | Bottom 25% | total |
| SMB | Top 25% | 9.21% (6.25%) | 12.55% (12.5%) | 3.35% (6.25%) | 25% |
| | Middle 50% | 9.62% (12.5%) | 25.94% (25%) | 14.23% (12.5%) | 50% |
| | Bottom 25% | 6.28% | 11.30% | 7.53% | 25% |

| | | | | | |
|---|---|---|---|---|---|
| | | (6.25%) | (12.5%) | (6.25%) | |
| | Total | 25% | 50% | 25% | 100% |

For the test of independence, the p value is 0.05, therefore, we reject the null hypothesis that SMB and GDP growth innovation are independent at a 10% significance level. From table 3, we also observe that SMB and GDP groweth innovation tend to move in the same direction as each other. For example, when GDP growth innovation is in the top state, the observed probability that SMB is also in the top state is 9.21%, higher than the expected probability of 6.25%.

(1) UK quarterly data

Table 4 shows that almost all the macroeconomic variables are associated with each other based on a chi-square test, apart from the term spread and GDP growth innovation. Though SMB moves in line with most of the macroeconomic variables: GDP growth, inflation innovation, the 3m T-bill and the term spread, HML is only associated with the 3m T-bill. The lack of relationship between the variables might be because of the loss of valuable information in the data transforming process when treating variables as categorical variables.

*Table 4: P value of the chi-square test (UK)*

| Chi-square | SMB | HML | GDP growth | GDPg inno | Inflation Inno | 3mT-bill | Term Spread |
|---|---|---|---|---|---|---|---|
| SMB | 0.00 | 0.84 | 0.03 | 0.18 | 0.06 | 0.04 | 0.00 |
| HML | 0.84 | 0.00 | 0.73 | 0.84 | 0.84 | 0.03 | 0.12 |
| GDP growth | 0.03 | 0.73 | 0.00 | 0.00 | 0.03 | 0.00 | 0.06 |
| GDPg inno | 0.18 | 0.84 | 0.00 | 0.00 | 0.00 | 0.01 | 0.11 |
| Inflation inno | 0.06 | 0.84 | 0.03 | 0.00 | 0.00 | 0.00 | 0.03 |
| 3mT-bill | 0.04 | 0.03 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| Term Spread | 0.00 | 0.12 | 0.06 | 0.11 | 0.03 | 0.00 | 0.00 |

*Ps. The null hypothesis is that categorical variable X and variable Y are independent. A p value lower than 0.10 indicates that we cannot reject the null hypothesis at a 10% significance level. Therefore, X and Y are associated with each other.*

(2) US quarterly data

From Table 5, we reject the null hypothesis of independence in most of the cases at a significance level of 0.10. The small p values between SMB and term spread suggest that term spread is a useful factor for explaining the size premium. The HML portfolio is associated with credit spread with a p value of 0.07, suggesting a strong relationship between credit spread and the value premium. Surprisingly, both SMB and HML portfolio returns are not associated with GDP growth.

*Table 5: P value of the Chi-square test (US)*

| Chi-square | SMB | HML | GDP growth | GDPg inno | Inflation Inno | 3mT-bill | Term Spread | Credit Spread |
|---|---|---|---|---|---|---|---|---|
| SMB | 0.00 | 0.22 | 0.49 | 0.05 | 0.13 | 0.64 | 0.05 | 0.22 |
| HML | 0.22 | 0.00 | 0.56 | 0.81 | 0.26 | 0.02 | 0.23 | 0.07 |
| GDP growth | 0.49 | 0.56 | 0.00 | 0.00 | 0.36 | 0.03 | 0.07 | 0.00 |
| GDPg inno | 0.05 | 0.81 | 0.00 | 0.00 | 0.37 | 0.41 | 0.04 | 0.02 |
| Inflation inno | 0.13 | 0.26 | 0.36 | 0.37 | 0.00 | 0.00 | 0.35 | 0.02 |
| 3mT-bill | 0.64 | 0.02 | 0.03 | 0.41 | 0.00 | 0.00 | 0.00 | 0.00 |
| Term Spread | 0.05 | 0.23 | 0.07 | 0.04 | 0.35 | 0.00 | 0.00 | 0.00 |
| Credit Spread | 0.22 | 0.07 | 0.00 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 |

What we are more interested in is whether the return premia and macroeconomic variables are associated with each other. Table 6 clearly shows that the state variables we chose indeed do provide information for the return premia.

*Table 6:Summary of Coincidence tests between return premia and macroeconomic variables*

| | UK | | US | |
|---|---|---|---|---|
| | SMB | HML | SMB | HML |
| Contemporaneous GDP growth | Correlated p=0.03* | Independent | Independent | Independent |
| GDP growth innovation | Independent | Independent | Correlated p=0.05* | Independent |
| Inflation Innovation | Correlated p=0.06* | Independent | Independent | Independent |

| | | | | |
|---|---|---|---|---|
| 3m T-bill | Correlated p=0.04* | Correlated p=0.03* | Independent | Correlated p=0.02* |
| Term Spread | Correlated p=0.00* | Independent | Correlated p=0.05* | Independent |
| Credit Spread | N/A | N/A | Independent | Correlated p=0.07* |

***III.ii SMB/HML portfolio returns analysis at different economic states***

Though the contingency table can show us whether a particular pair of variables is independent or not, it cannot provide clear information about the direction of their movements. Therefore, we treat all the variables as quantitative variables and relate portfolio returns to the economic environment by examining the returns on the HML and SMB portfolios in different economic states.

Table 8 displays a summary of the SMB/HML portfolios in different economic states. In order to better understand the summary table, we will illustrate the case for GDP growth first.

As states of the world can be discrete, we characterize states that exhibit the highest 25% of economic growth as “top states”, the middle 50% as “normal states” and the lowest 25% as “bottom states”. In tables 7.1 and 7.2, we calculate the average returns to the HML and SMB portfolios in different states of GDP growth for the current quarter and the next quarters respectively. The differences between the average returns in the top states and bottom states are reported, and so are the t-statistics for their differences. Here we assume that the average returns in the top and bottom states are independently and normally distributed for the purpose of t-statistics.

Both value and small caps have performed best in periods of higher GDP growth (positive t-statistics) – a result that accords with intuition, as one would expect economic growth to provide increased investment opportunities and to lower the risk for distressed companies. Comparing tables 7.1 and 7.2, we find that using the next period’s GDP growth rate as an explanatory variable has an advantage over contemporaneous GDP growth, which is consistent with the fact that asset prices are forward looking. Compared with the chi-square independence test, this approach proves a much stronger relationship between SMB/HML and GDP growth. In addition, we observe that, regardless of the country, the size premium is always more significant than the value premium.

*Table 7.1: The performance of the HML, SMB zero-investment portfolios in different states of the economy (contemporanous)*

| Zero-investment portfolio returns sorted by contemporaneous GDP growth | | | | | | |
|---|---|---|---|---|---|---|
| Country | Portfolio | Top states | Normal | Bottom states | Difference between top and bottom states | Independent two sample (top states and bottom states) t-test |
| UK | HML | 1.04% | 1.94% | 1.08% | -0.04% | -0.04 |
| | SMB | 3.25% | -0.14% | -0.16% | 3.41% | 3.02* |
| | GDP | 1.76% | 0.64% | -0.55% | 2.31% | 15.76 |
| US | HML | 1.52% | 0.95% | 0.53% | 0.98% | 0.805 |
| | SMB | 1.35% | 0.41% | 0.11% | 1.24% | 1.258 |
| | GDP | 2.05% | 0.81% | -0.36% | 2.41% | 21.40 |

*Table 7.2: The performance of the HML, SMB zero-investment portfolios in different states of the economy (next period's GDP growth)*

| Zero-investment portfolio returns sorted by next period's GDP growth | | | | | | |
|---|---|---|---|---|---|---|
| Country GDP (t+1) | Portfolio (t) | Top states | Normal | Bottom states | Difference between top and bottom states | Independent two sample (top states and bottom states) t-test |
| UK | HML | 1.63% | 1.73% | 1.02% | 0.61% | 0.67 |
| | SMB | 2.66% | 0.14% | 0.04% | 2.62% | 2.08* |
| US | HML | 1.40% | 1.53% | -0.35% | 1.74% | 1.59 |
| | SMB | 1.20% | 0.82% | -0.52% | 1.72% | 1.77* |

The number in each cell of table 8 is the t-statistic for the difference between the average returns in the top state and bottom state, where states are defined by the chosen macroeconomic variables. For example, the numbers in the third row, 1.9, 0.7, 1.3 and -0.4, respectively, suggest that SMB/HML are likely to perform well when GDP growth innovation is high. Comparing the results from GDP growth and GDP growth innovation, we find that the use of GDP series conferred a discernable advantage over the unexpected economic growth. As expected, we observed that the unexpected inflation has a negative relationship with SMB and a positive relationship with HML. The results for short term interest rate are also the same as expected, with value and smaller stocks performing better when short term interest rates are low. Such firms will be more likely to suffer (compared with longer duration growth stocks) when short term rates are high.

Consistent with the results from the chi-square independence test, there is always a positive relationship between SMB/HML and the term spread. In addition, we find that the term spread effect is much stronger in the UK than the US. A positive relationship between HML/SMB and the credit spread is also identified, especially for the SMB portfolio, suggesting that size factor is more likely to be related to asset distressed risk.

*Table 8: T-statistic summary of the SMB/HML portfolio returns analysis at different economic states*

| T-statistics | UK | | US | |
|---|---|---|---|---|
| | SMB | HML | SMB | HML |
| Contemporaneous GDP growth | +3.0* | -0.0 | +1.3 | +0.8 |
| Next quarter's GDP growth | +2.1* | +0.7 | +1.8* | +1.6 |
| GDP growth innovation | +1.9* | +0.7 | +1.3 | -0.4 |
| Inflation Innovation | -1.6 | +0.2 | -2.2* | +1.0 |
| 3m T-bill | -3.2* | -1.8* | -2.1* | +0.1 |
| Term Spread | +3.0* | +2.3* | +1.0 | +1.2 |
| Credit Spread | N/A | N/A | +2.6* | +0.6 |

## IV. Threshold Regression Analysis

### IV.i. Methodology

We used threshold regression to construct a portfolio return regression, in order to capture a possible non-linearity effect. The threshold model allows us to split the sample into different regimes. To our knowledge, this is the first paper that analyses style factor returns with Hansen's threshold regression model.

The threshold regression (TR) model is given by:

$$R_t = \beta_1^{'} x_t + \varepsilon_t \qquad \text{When } q_t \le c \tag{1}$$

$$R_t = \beta_2^{'} x_t + \varepsilon_t \qquad \text{When } q_t > c \tag{2}$$

Where $R_t$ is the portfolio returns, $x_t$ is a set of explanatory variables, $\beta_1^{'}$ and $\beta_2^{'}$ are the corresponding coefficient vectors, $\varepsilon_t$ is the regression error, $q_t$ is the threshold variable which can be some function of $x_t$, $c$ is the unknown threshold level. In our model, we set $q_t = x_t$. The threshold variable $q_t$ is used to split the sample into different regimes. A n-regime threshold regression allows the coefficient vectors to change *n* times based on the value of the threshold variable $q_t$. If there is no threshold effect, in the case of 2 regimes, then $\beta_1 = \beta_2$.

We can also express equations (5) and (6) within a single equation with the implementation of the dummy variables $d_t(c) = I(q_t \le c)$, where $I$ is the indicator function: $d_t(c) = I(q_t \le c) = 1$ if $q_t \le c$; $d_t(c) = I(q_t \le c) = 0$ if $q_t > c$

$$R_t = \beta \ x_t + \phi \ x_t \ d_t(c) + \varepsilon_t = \beta \ x_t + \phi \ x_t(c) + \varepsilon_t \tag{3}$$

Here $\beta = \beta_2$. Equation (3) allows different regression coefficients across different regimes.

From a computational standpoint, we express equation (3) in matrix form. We define the t by 1 vectors $R$ and $\varepsilon$ by stacking the variables $R_t$ and $\varepsilon_t$, and the t by I matrix $X$ and $X_c$ by stacking $x_t$ and $x_t(c)$. Then equation (3) can be expressed as

$$R = \beta \ X + \phi \ X_c + \varepsilon \tag{4}$$

We can estimate the parameters { $\beta$ , $\phi$ , $c$ } by using least squares estimation, minimizing the objective function.

$$S(\beta,\phi,c) = (R - \beta X - \phi X_c)' \ (R - \beta X - \phi X_c) \tag{5}$$

Hansen (2000) developed an algorithm based on a sequential OLS estimation, which extensively searches the threshold level $c$ over all values $c = q_t$. This iterative estimation starting from some initial guess of $c$, provides estimates of { $\beta$ , $\phi$ , $c$ } which minimize the objection function. Such a procedure is known to be consistent.

We implement the heteroscedasticity-consistent Lagrange multiplier (LM) test to test the null hypothesis that there is no threshold effect $\beta_1 = \beta_2$. As the threshold level $c$ is not identified under the null of no threshold effect, in the testing stage, we need to implement a bootstrap method to calculate the p values. Hansen (1996) showed that this bootstrap procedure generates asymptotically correct p values.

***IV.ii Univariate regression***

(1) US SMB portfolio

In this section, we run the univariate regression of the US SMB portfolio returns against each individual macroeconomic variable. Table 9 displays the F-test statistics on the threshold effect for the US SMB portfolio and different macroeconomic variables. For example, using 10,000 bootstrap replications, the p-value for the threshold model using contemporaneous GDP growth as the threshold variable was marginally significant at 0.044. This suggests that there is a threshold effect. Figure 1 shows the likelihood ratio LR(c) as a function of the threshold level c for US GDP growth. The estimated value of the threshold level c is at c=0.017481 which gives the minimum of the LR. The asymptotic 95% critical value is also plotted. These results suggest that a two-regime specification is appropriate in the model of US SMB against the contemporaneous GDP growth. There are 203 data points below and 40 above the threshold level of GDP growth at 0.017481.

From the bootstrap p value in table 9.1, the null hypothesis of linearity is only rejected when the threshold variable is GDP growth or next quarter's GDP growth rate, where the threshold estimates are 0.017 and 0.0076, the p values are 0.044 and 0.092 respectively. For other macroeconomic variables, the p values are all higher than 0.10, in favour of no threshold effect. Therefore, among all the models, only GDP growth (contemporaneous or next period) is selected as the appropriate threshold variable. This is consistent with the notion that GDP growth is one of the best proxies for describing the economic environment. Other variables, for example inflation alone, are unlikely to give as complete a picture of the economic state.

*Table 9.1: F-test of no threshold (H0: $\beta_1 = \beta_2$) for the model of US SMB portfolio against each individual macroeconomic variable*

| | Threshold Estimate c | LM test for no threshold | Bootstrap p value |
|---|---|---|---|
| GDP growth | 0.017481 | 10.337 | 0.044* |
| Next quarter's GDP growth | 0.007657 | 9.061 | 0.092* |
| Inflation inno | -0.0039 | 3.8509 | 0.719 |
| 3m T-bill | 0.0025 | 5.9996 | 0.487 |
| Term spread | -0.0075 | 4.143 | 0.799 |
| Credit spread | 0.0073 | 7.038 | 0.256 |

*Figure 1: The likelihood ratio LR(c) as a function of the threshold level c for US SMB and contemporaneous GDP growth*

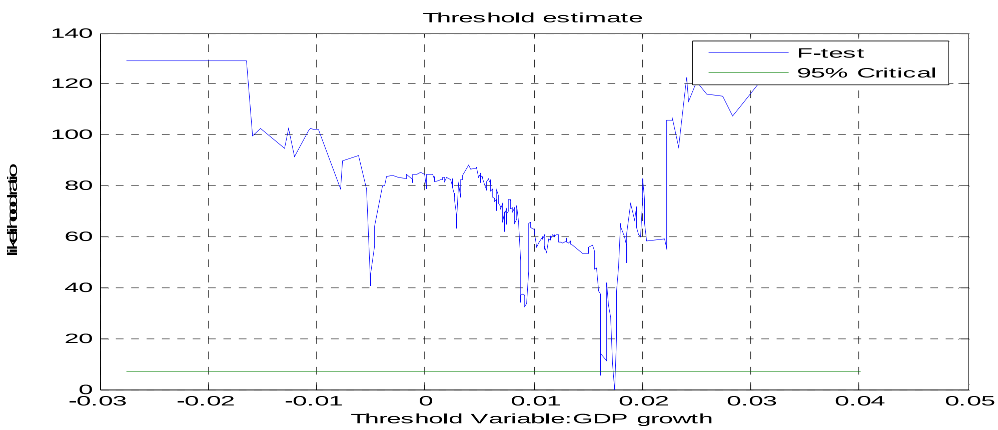


*Figure 2: The likelihood ratio LR(c) as a function of the threshold level c for US SMB and next quarter's GDP growth*

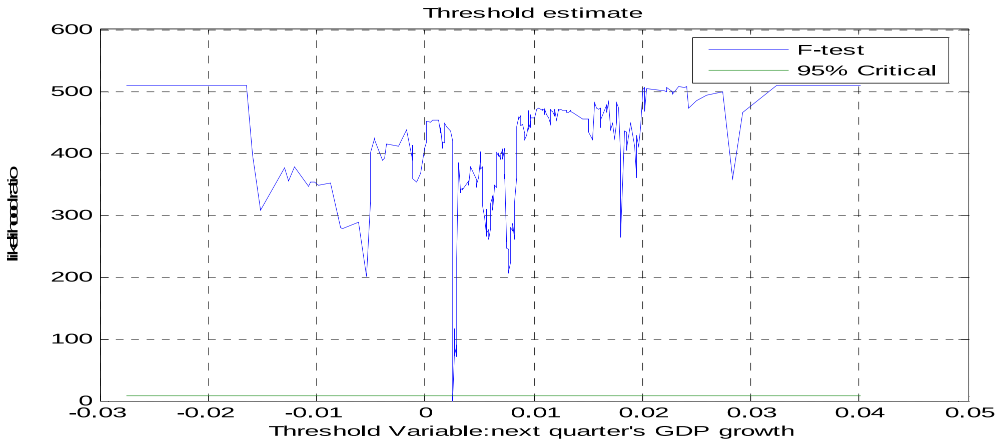


Table 9.2 lists the results of the univariate OLS regression for the global sample and two univariate OLS regressions for the two separate regimes. It contains information for 18 univariate regressions. The samples are split according to the estimated threshold level $c$ which minimizes the likelihood ratio. The coefficients in the global OLS regression have the same sign as we expected and observed in the previous sections.

In the model of contemporaneous GDP growth, insignificant coefficients and low R-squared coefficients are observed in all three different sets of regressions. We are therefore in favour of the model using next quarter's GDP growth as the independent variable and threshold variable. In this case, the sample is evenly split. Significant positive coefficients and relatively higher R-squared results are observed in both subsamples. Thus small cap firms perform better in periods of higher expected GDP growth which is consistent with the results in discrete state analysis. Also, SMB has a significant positive correlation with the term spread, which further confirms our conclusion in the discrete analysis. Interestingly, SMB and credit spreads are positively associated when the credit spread is higher than 0.0073; and negatively related when the credit spread is lower than 0.0073. Generally, from table 9.2, we observe that the goodness of fit of the model improves after dividing the full sample into two different regimes.

*Table 9.2: Univariate OLS regression for the global sample and two univariate OLS regressions for the two separate subsamples*

| | Global without threshold | q<=c | q>c |
|---|---|---|---|

| | Coef | T ratio | R-squared | Coef | T ratio | R-squared | Coef | T ratio | R-squared |
|---|---|---|---|---|---|---|---|---|---|
| GDP growth | 0.178 | 0.477 | 0.001 | -0.892 | -1.777 | 0.016 | -0.069 | -0.069 | 0.000 |
| Next quarter's GDP growth | 0.728 | 2.154* | 0.017 | 2.399 | 3.198* | 0.072 | 1.358 | 2.144* | 0.034 |
| Infl inno | 0.024 | 0.055 | 0.000 | 18.686 | 3.228* | 0.518 | 0.258 | 0.590 | 0.002 |
| 3m T-bill | -0.136 | -1.291 | 0.006 | 38.416 | 1.940 | 0.117 | -0.268 | 0.142 | 0.002 |
| Term spread | 0.680 | 2.069* | 0.021 | 7.316 | 3.658* | 0.716 | 0.988 | 2.717* | 0.037 |
| Credit spread | 0.625 | 1.106 | 0.006 | -41.330 | -5.32* | 0.603 | 0.927 | 1.427 | 0.011 |

(2) US HML portfolio

The following section adopts the same procedure, as shown above, on the US HML portfolio. Different from SMB, the null hypothesis of linearity is only rejected in favour of a threshold effect when the threshold variable is the 3m T-bill or term spread. The p values are 0.0702 and 0.0782, respectively. However, we find that these two threshold variables could not evenly split the whole sample. For example, there are 280 data points above and 17 below the threshold level of the 3m T-bill at 0.0014. Unlike the case for the US SMB portfolio, the use of next quarter's GDP growth in the threshold regression cannot reject the null hypothesis of no threshold effect. This is consistent with the finding in discrete analysis that GDP growth is more closely related with the SMB than the HML portfolio.

The coefficients' signs in the full sample regression are mostly consistent with our expectations, though all  are not significant. Table 10 also displays the OLS regression results for two subsamples split by the threshold level estimate. The model giving us the better fit is the one with the term spread as the explanatory variable; where we observe that growth firms with a greater dependence on the long term rate suffer more when the yield curve is steepest. Comparing goodness of fit of the different models, clearly the R-squared coefficients increase and the coefficients become more significant after splitting the whole sample.

*Table 10: Univariate OLS regression for the full sample and two univariate OLS regressions for the two separate regimes*

| | Global without threshold | | | q<=c | | | q>c | | |
|---|---|---|---|---|---|---|---|---|---|
| | Coef | T ratio | R-squared | Coef | T ratio | R-squared | Coef | T ratio | R-squared |
| GDP growth | 0.379 | 0.933 | 0.004 | -0.184 | -0.337 | 0.0007 | -2.593 | -1.307 | 0.077 |
| Next quarter's GDP growth | 0.472 | 1.401 | 0.007 | -0.357 | -0.262 | 0.001 | -0.341 | -0.702 | 0.002 |
| Infl inno | 0.313 | 0.781 | 0.002 | -0.946 | -1.299 | 0.007 | -1.710 | -1.354 | 0.035 |
| 3m T-bill | -0.049 | -0.388 | 0.001 | 107.49 | 2.47* | 0.313 | -0.076 | -0.577 | 0.002 |
| Term spread | 0.482 | 1.548 | 0.010 | 4.895 | 3.31* | 0.356 | 0.908 | 2.563* | 0.027 |
| Credit spread | -0.029 | -0.054 | 0.000 | -0.611 | -0.931 | 0.004 | -13.8 | -2.715 | 0.214 |

(3) UK SMB portfolio

For the UK SMB portfolio, the null hypothesis of linearity is only rejected when the threshold variable is inflation, with a p-value of 0.687. Surprisingly, in contrast to the US SMB portfolio case, neither contemporaneous GDP growth nor next quarter's GDP growth shows evidence of a threshold effect. This is contrary to the results from the discrete state analysis, where the UK SMB portfolio is significantly related to GDP growth. The coefficient signs in the regression are significant, and also consistent with our expectations. Table 11 also displays the regression results for two subsamples split by the threshold level estimate.

*Table 11: Univariate OLS regression for the full sample and two univariate OLS regression for the two separate regimes*

| | Global without threshold | | | Q<=c | | | q>c | | |
|---|---|---|---|---|---|---|---|---|---|
| | Coef | T ratio | R-squared | Coef | T ratio | R-squared | Coef | T ratio | R-squared |
| GDP growth | 0.997 | 2.537* | 0.026 | 0.223 | 0.326 | 0.001 | -0.767 | -1.159 | 0.013 |
| Next quarter's GDP growth | 0.925 | 1.962* | 0.022 | -18.51 | -1.60* | 0.375 | 0.648 | 1.589* | 0.010 |
| Infl inno | -0.689 | -1.65* | 0.025 | -2.840 | -2.23* | 0.044 | -1.467 | -2.34* | 0.104 |
| 3m T-bill | -0.422 | -2.93* | 0.045 | -2.169 | -2.69* | 0.095 | -0.819 | -2.64* | 0.082 |
| Term spread | 0.8142 | 2.893* | 0.0627 | 2.378 | 4.988* | 0.230 | 0.923 | 1.399* | 0.029 |

(4) UK HML portfolio

The threshold effect regression for the UK HML portfolio does not provide any insight. From the p-value, all the threshold variables cannot reject the null hypothesis of no threshold effect. None of the regressions in table 12, apart from the term spread, generate significant coefficients. Again, in contrast to the US case, GDP growth does not offer any help in dividing the whole sample. In fact, in the state analysis, the UK HML portfolio also shows a very weak relationship with GDP growth.

*Table 12: Univariate OLS regression for the full sample and two univariate OLS regressions for the two separate regimes*

| | Global without threshold | | | q<=c | | | q>c | | |
|---|---|---|---|---|---|---|---|---|---|
| | Coef | T ratio | R-squared | Coef | T ratio | R-squared | Coef | T ratio | R-squared |
| GDP growth | 0.251 | 0.879 | 0.003 | 0.651 | 1.379 | 0.011 | 1.195 | 1.388 | 0.0344 |
| Next quarter's GDP growth | 0.360 | 1.350 | 0.006 | 0.298 | 0.824 | 0.002 | -2.578 | -0.825 | 0.219 |
| Infl inno | 0.043 | 0.203 | 0.000 | -4.049 | -1.86* | 0.063 | 0.048 | 0.184 | 0.0002 |
| 3m T-bill | -0.165 | -1.564 | 0.012 | 0.719 | 0.488 | 0.007 | 0.080 | 0.465 | 0.002 |
| Term spread | 0.433 | 2.739* | 0.031 | -0.372 | -0.650 | 0.004 | -0.356 | 0.550 | 0.002 |

### *IV.iii Multiple regression*

We now turn to a multiple regression analysis of style factor portfolios against all the macroeconomic variables. As can be seen from the previous analysis, there is not yet a variable which has won dominance in splitting the sample into two different regimes. Therefore, we have to search for the threshold variable from among all the explanatory variables and choose the one with the lowest p-value as the first threshold variable q1. After splitting the sample into two regimes based on the selected threshold variable q1, we again perform the same analysis on each subsample, and verify whether there is a second threshold effect within each subsample. This regression tree methodology tests whether the portfolio returns are influenced by initial conditions of multilayer threshold variables.

(1) US SMB portfolio

We need to search the five potential threshold variables, and verify that there is indeed threshold effect. Table 13.1 shows the p-value using different macroeconomic variables as the threshold variable in the multivariate regressions. The use of the 3-month T-bill as the threshold variable gives us the lowest p-value of 0.0261, suggesting that the linearity hypothesis is rejected with a significance level of 2.61%. Therefore we can split the sample into two regimes, where there are 180 data points below and 30 above the threshold level of the 3m T-bill at 0.07920. Table 13.2 displays the multivariate OLS regression of the SMB portfolio against all the macroeconomic variables. Most of the coefficient signs are consistent with our expectations, though most of them are only marginally significant. It is clear that the model has improved after the sample splitting, in terms of both R-squared and t-statistics.

After choosing the 3m T-bill as the first threshold variable and fixing the threshold level at the estimated level 0.07920, we can then follow the same procedure to each subsample for a second threshold effect. The second sample split test results are displayed in table 13.3. Among the 184 observations with the 3m T-bill lower than 0.0792, various variables are searched to identify the one giving the lowest p-value in this subsample. A second sample split based on term spread gives the lowest p-value of 0.04, which divides the subsample into a further two regimes. For the sample with a 3m T-bill higher than 0.0790, we could not observe any significant p-value, in favour of no threshold effect in that subsample. The results suggest a three-regime model for the US SMB portfolio. It is worth noticing that the sign for the credit spread is ambiguous.

*Table 13.1: F-test of no threshold (H0: $\beta_1 = \beta_2$) for the multivariate model of US SMB*

| | Threshold Estimate c | LM test for no threshold | Bootstrap p value |
|---|---|---|---|
| Next quarter's GDP growth | -0.004687 | 12.163 | 0.3254 |
| Infl inno | 0.017341 | 12.043 | 0.3236 |
| 3m T-bill | 0.07920 | 17.6588 | 0.0261* |
| Term spread | 0.0037 | 11.2455 | 0.4326 |
| Credit spread | 0.0238 | 16.1870 | 0.0539 |

*Table 13.2: Multiple OLS regression for the full sample and two multiple OLS regressions for the two subsamples*

| | Global without threshold | | Subsample regression where 3mT-bill<=0.0792 | | Subsample regression where 3mT-bill>0.0792 | |
|---|---|---|---|---|---|---|
| Explanatory variables | Coef | T ratio | Coef | T ratio | Coef | T ratio |
| Constant | -0.01 | -1.08 | 0.006 | 0.48 | 0.0003 | 0.006 |
| Next quarter's GDP growth | 0.83 | 1.83* | 0.42 | 0.83 | 1.21 | 1.39 |
| Infl inno | 0.79 | 1.19 | 0.21 | 0.26 | 2.23 | 1.53 |
| 3m T-bill | -0.15 | -1.24 | -0.38 | -1.74 | -1.23 | -2.54* |
| Term spread | 0.45 | 1.23 | 1.07 | 2.30* | -0.68 | -1.12 |
| Credit spread | 0.56 | 0.82 | -0.23 | -0.31 | 5.11 | 3.67* |
| R-squared | 0.0419 | | 0.0695 | | 0.405 | |
| Number of Obs | 214 | | 184 | | 30 | |

*Table 13.3: Second sample split test: F-test of no threshold effect (H0: $\beta_1 = \beta_2$) for the two subsamples*

| Variables | Bootstrap p value for the subsample where 3m T-bill <=0.0792 | Bootstrap p valuefor the subsample where 3m T-bill >0.0792 |
|---|---|---|
| Next quarter's GDP growth | 0.55 | 0.756 |
| Infl inno | 0.09 | 0.199 |
| 3m T-bill | 0.16 | 0.197 |
| Term spread | 0.04* | 0.388 |
| Credit spread | 0.24 | 0.449 |

*Table 13.4: OLS regression for different regimes in the subsample 3m T-bill<=0.0792*

| | First subsample where 3m T-bill <=0.0792 without threshold | | Subsample regression where 3m T-bill <=0.0792 and term spread<=0.0028 | | Subsample regression Where 3m T-bill <=0.0792 and term spread>0.0028 | |
|---|---|---|---|---|---|---|
| Explanatory variables | Coef | T ratio | Coef | T ratio | Coef | T ratio |
| Constant | 0.006 | 0.48 | 0.16 | 3.58* | -0.008 | -0.68 |
| Next quarter's GDP growth | 0.42 | 0.83 | 0.50 | 0.40 | 0.15 | 0.26 |
| Infl inno | 0.21 | 0.26 | -0.63 | -0.30 | 0.86 | 1.01 |
| 3m T-bill | -0.38 | -1.74* | -1.84 | -2.07* | -0.42 | -1.81* |
| Term spread | 1.07 | 2.30* | 5.94 | 1.98* | 0.80 | 1.46 |
| Credit spread | -0.23 | -0.31 | -3.63 | -2.78* | 0.69 | 0.81 |
| R-squared | 0.0695 | | 0.497 | | 0.064 | |
| Number of Obs | 184 | | 28 | | 156 | |

(2) US HML portfolio

We performed the same analysis on the US HML portfolio. From table 14.1, three out of the five potential threshold variables support a threshold effect, with next quarter's GDP growth generating the lowest p value of 0.009, in favour of the threshold effect. Therefore we can split the sample into two regimes, where there are exactly 107 data points below and 107 above the threshold level of next quarter's GDP growth at 0.00789. Table 14.2 displays the multivariate OLS regression of the HML portfolio against all the macroeconomic variables. The coefficient signs for the full sample regression are the same as expected, though not significant. By dividing it into two samples using next quarter's GDP growth, the coefficients for next quarter's GDP growth, term spread and credit spread become significant, also with a higher R-squared.

After choosing GDP growth as the first threshold variable and fixing the threshold level at the estimated level 0.00789, we can then follow the same procedure within each subsample for a second threshold effect. The second sample split test results are displayed in table 14.3. For the subsample with the next quarter's GDP growth smaller than 0.00789, the credit spread is identified as the second threshold variable giving the lowest p-value, which divides the subsample further into two regimes. For the subsample with next quarter's GDP growth higher than 0.0790, term spread is chosen as the third threshold variable, splitting that subsample into two regimes. Therefore, altogether we have four regimes for the US HML portfolio. The regression results for the regimes are listed in table 14.4 and 14.5. As with the US SMB portfolio, there is no consistent sign for the credit spread.

*Table 14.1: F-test of no threshold (H0: $\beta_1 = \beta_2$) for the multivariate model of US HML*

| | Threshold Estimate c | LM test for no threshold | Bootstrap P value |
|---|---|---|---|
| Next quarter's GDP growth | 0.00789 | 19.1345 | 0.009* |
| Infl inno | 0.0054 | 14.4421 | 0.101 |
| 3m T-bill | 0.0426 | 18.2909 | 0.010* |
| Term spread | 0.0062 | 15.9293 | 0.056* |
| Credit spread | 0.0089 | 12.9753 | 0.231 |

*Table 14.2: Global multivariate OLS regression of HML against all the macro variables and two multiple OLS regressions for the two subsamples*

| | Global without threshold | | Subsample regression where next quarter's GDP growth<=0.00789 | | Subsample regression where next quarter's GDP growth>0.00789 | |
|---|---|---|---|---|---|---|
| Explanatory variables | Coef | T ratio | Coef | T ratio | Coef | T ratio |
| Constant | 0.00 | 0.20 | -0.03 | -2.16* | 0.01 | 0.72 |
| Next quarter's GDP growth | 0.33 | 0.77 | 1.75 | 2.25* | 1.04 | 1.37 |
| Infl inno | 0.27 | 0.40 | 0.67 | 0.78 | -1.01 | -1.22 |
| 3m T-bill | 0.03 | 0.15 | 0.02 | 0.09 | 0.26 | 0.92 |
| Term spread | 0.58 | 1.18 | -0.44 | -0.61 | 1.61 | 2.95* |
| Credit spread | -0.39 | -0.45 | 1.96 | 1.67* | -2.94 | -2.87* |
| R-squared | 0.014 | | 0.069 | | 0.177 | |
| Number of Obs | 214 | | 107 | | 107 | |

*Table 14.3: Second sample split test: F-test of no threshold effect (H0: $\beta_1 = \beta_2$) for the two subsamples*

| | Bootstrap p value for the subsample where next quarter's GDP growth<=0.00789 | Bootstrap p value for the subsample where next quarter's GDP growth>0.00789 |
|---|---|---|
| Next quarter's GDP growth | 0.0560* | 0.0120 |
| Infl inno | 0.3040 | 0.6000 |
| 3m T-bill | 0.4590 | 0.1500 |
| Term spread | 0.4390 | 0.0050* |
| Credit spread | 0.0410* | 0.6320 |

*Table 14.4: OLS regression for different regimes in the subsample next quarter's GDP growth<=0.00789*

| | First subsample where next quarter's GDPgrowth<=0.00789, without threshold | | Subsample regression where next quarter's GDP growth<=0.00789 and credit spread<=0.0211 | | Subsample regression where next quarter's GDP growth<=0.00789 and credit spread>0.0211 | |
|---|---|---|---|---|---|---|
| Explanatory variables | Coef | T ratio | Coef | T ratio | Coef | T ratio |
| Constant | -0.03 | -2.16* | 0.01 | 0.51 | 0.13 | 1.83* |
| Next quarter's GDP growth | 1.75 | 2.25* | 0.30 | 0.44 | 6.71 | 3.01* |
| Infl inno | 0.67 | 0.78 | 2.74 | 2.78* | -0.37 | -0.29 |
| 3m T-bill | 0.02 | 0.09 | -0.28 | -0.93 | 0.54 | 1.94* |
| Term spread | -0.44 | -0.61 | 0.87 | 1.11 | -1.79 | -1.89* |
| Credit spread | 1.96 | 1.67* | -2.34 | -1.20 | -3.10 | -1.16 |
| R-squared | 0.0689 | | 0.1010 | | 0.3441 | |
| Number of Obs | 107 | | 75 | | 32 | |

*Table 14.5: OLS regression for different regimes in the subsample next quarter's GDP growth>0.00789*

| | First subsample Where next quarter's GDP growth>0.00789 without threshold | | Subsample regression Where next quarter's GDP growth>0.00789 And term spread<=0.0028 | | Subsample regression Where next quarter's GDP growth>0.00789 and term spread>0.0028 | |
|---|---|---|---|---|---|---|
| Explanatory variables | Coef | T ratio | Coef | T ratio | Coef | T ratio |
| Constant | 0.01 | 0.72 | 0.06 | 1.61* | -0.04 | -1.42 |
| Next quarter's GDP growth | 1.04 | 1.37 | -3.83 | -2.78* | 3.31 | 3.93* |
| Inflation | -1.01 | -1.22 | -2.94 | -1.70* | -1.60 | -1.79* |
| 3m T-bill | 0.26 | 0.92 | 1.87 | 2.82* | -0.07 | -0.24 |
| Term spread | 1.61 | 2.95 | 3.52 | 2.49* | 3.08 | 2.77* |
| Credit spread | -2.94 | -2.87 | -6.82 | -3.33* | -2.77 | -2.59* |
| R-squared | 0.178 | | 0.437 | | 0.286 | |
| Number of Obs | 107 | | 42 | | 65 | |

*(3) UK SMB portfolio*

Following the same procedure, we test the threshold effect for the UK SMB portfolio. Table 15.1 shows the p-value using different macroeconomic variables as the threshold variable in the multivariate regression. The p-value for the threshold model using next quarter's GDP growth as the threshold variable is marginally significant at 0.0980. However, only 7 observations have next quarter's GDP growth lower than the threshold level -0.01225, leaving the other 144 observations in the other regime. This is contradictory to our restriction that each subsample should account for at least 10% of the total sample size.  For the other variables most likely to have significant p-value, we observe that the sample size is also not close to evenly split. Therefore, the threshold regression model is not appropriate for the UK SMB portfolios. Table 15.1 displays the multiple regression of the UK SMB portfolio against all the macroeconomic variables without considering the threshold effect. The model generates an R-squared of 12.83% and significant coefficients for inflation and the term spread.

*Table 15.1: F-test of no threshold (H0: $\beta_1 = \beta_2$) for the multivariate model of UK SMB*

| | Threshold level c | LM test for no threshold | Bootstrap p value |
|---|---|---|---|
| Next quarter's GDP growth | -0.01225 | 13.0221 | 0.0980* |
| Infl inno | 0.0470 | 12.6833 | 0.1310 |
| 3m T-bill | 0.0493 | 10.4426 | 0.3220 |
| Term spread | 0.0357 | 12.4217 | 0.1400 |

*Table 15.2 Multiple regression of UK SMB portfolio against all the macroeconomic variables without considering the threshold effect*

| | Coefficient | t-stat |
|---|---|---|
| Constant | 0.0086 | 0.5030 |
| Next quarter's GDP growth | 0.7858 | 1.3838 |
| Infl inno | -1.0152 | -2.3012* |
| 3m T-bill | 0.0475 | 0.2256 |
| Term spread | 0.9323 | 3.1010* |
| R-squared | 12.83% | |

*(4) UK HML portfolio*

We performed the same analysis for UK HML portfolios. Table 16.1 shows the p-values using different macroeconomic variables as the threshold variable in the multivariate regression. All the variables generate insignificant p-values, indicating no threshold effect. Therefore, we cannot reject the null hypothesis of no threshold effect for the UK HML portfolio. Table 16.2 displays the multiple regression of the UK HML portfolio against all the macroeconomic variables without considering the threshold effect. The goodness of fit of this model is much worse compared with the UK SMB portfolio case.

*Table 16.1: F-test of no threshold (H0: $\beta_1 = \beta_2$) for the multivariate model of UK HML*

| | LM test for no threshold | Bootstrap p value |
|---|---|---|
| Next quarter's GDP growth | 10.5832 | 0.2840 |
| Infl inno | 6.8167 | 0.8940 |
| 3m T-bill | 11.2279 | 0.2080 |
| Term spread | 9.7256 | 0.3900 |

*Table 16.2 Multiple regression of UK HML portfolio against all the macroeconomic variables without considering the threshold effect*

| | Coefficient | t-stat |
|---|---|---|
| constant | 0.0205 | 1.5091 |
| Next quarter's GDP growth | 0.1503 | 0.3337 |
| Infl inno | 0.1724 | 0.4928 |
| 3m T-bill | -0.1431 | -0.8563 |
| Term spread | 0.3044 | 1.2766 |
| R-squared | 0.0354 | |

We find that the threshold regression approach, for both univariate and multivariate regressions, works more effectively in the US than in the UK. This is largely attributed to a much longer data history available for the US, which provides us with more economic cycles for study. In addition, the threshold regression analysis reinforces our findings in the discrete state analysis that value and small caps have performed best in periods of higher GDP and that value and small stocks perform better when short term interest rates are low. In addition, there is always a positive relationship between SMB/HML and term spread, which is a proxy for asset duration risk.

## V. Conclusion

This paper established a significant link between macroeconomic state variables and style factors. Both value and small caps have performed best in periods of higher GDP growth – a result that accords with intuition, as one would expect economic growth to provide increased investment opportunities and to lower the risk for distressed companies. A negative relationship between SMB and unexpected inflation is more commonly expected, suggesting that small size firms suffer more in the unexpected inflation environment. On the other hand, there is more likely to be a positive relationship between unexpected inflation and the value premium. In addition, value and smaller stocks perform better when short term interest rates are low. Such firms will be more likely to suffer (compared to longer duration growth stocks) when short term rates are high. There is also always a positive relationship between SMB/HML and term spread, which is a proxy for asset duration risk.

This paper contributes to the literature by examining the economic nature of the Fama-French size and book-to-price factors. Other authors have also examined them but we have used more regime-based methods to try to identify the appropriate macroeconomic phenomena. We have chosen a set of economic variables as systematic risks on the stock market and uncovered the economic risk behind the size premium and value premium. Of course, we cannot exhaustively explore all the possible state variables, but the sets we have chosen perform well and the results confirm our economic prior expectations. Moreover, for several factors we investigated, (short term rates, inflation, term spread and credit spread) it appears that value stocks perform well during both high and low levels of the factors, perhaps indicating that distressed stocks gain in such extreme environments. These issues will be addressed in our future work.

## *References:*

Aretz, Bartram and Pope (2007), Macroeconomic Risks and Characteristic-Based Factor Models

Black, Mao and McMillan (2008), The Value Premium and Economic Activity: Long Run Evidence from the US

Chen, Roll and Ross (1986), Economic Forces and the Stock Market, The Journal of Business, Vol.59, No.3

Elroy Dimson, Stefan Nagel and Garrett Quigley (2003), Capturing the value premium in the UK, Financial Analysts Journal, 59(6): 35-45

Elroy Dimson, Paul Marsh and Mike Staunton (2008), Size and style effects over the long run, [in] Global Investment Returns Yearbook 2008,London: ABN Amro.

Fama and French (1992), Common Risk Factors in the Returns on Stocks and Bonds, Journal of Financial Economics, Vol. 33

Fama and French (1992), The Cross-Section of Expected Stock Returns, The Journal of Finance, Vol.47. No.2

Fama and French (1996), Multifactor Explanation of Asset Pricing Anomalies, The Journal of Finance, Vol.51, No.1

Ferson and Harvey (1991), The Variation of Economic Risk Premiums, The Journal of Political Economy, Vol.99, No.2

Liew and Vassalou (1999), Can Book-to-Market, Size and Momentum Be Risk Factors that Predict Economic Growth?

Maria Vassalou (2000), New Related to Future GDP Growth as Risk Factors in Equity Returns

Patrick Kelly (2003), Real and Inflationary Macroeconomic Risk in the Fama and French Size and Book-to-Market Portfolios

Robert C. Merton (1973), An Intertemporal Capital Asset Pricing Model, Econometrica, Vol.41, No.5

Ralitsa Petkova (2005), Do the Fama-French Factors Proxy for Innovations in Predictive Variables?